LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2023**

16/17/18UMT3MC02 – VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Part A

Answer ALL questions: **(10 x 2 = 20)**

1. Find $\nabla\emptyset$ at (x, y, z) if $\emptyset = x^3 + y^3 + 3xyz$.
2. Prove that $div\,(\vec{r}) = 3$, where $\vec{r}$ is the position vector.
3. Find a such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.
4. Define a conservative vector field.
5. If $\vec{F} = y\,\vec{i} - x\vec{j}$, evaluate $\int_C \vec{F}\,.\,d\vec{r}$ from $(0, 0)$ to $(1, 1)$ along the curve $y = x$.
6. State Green's theorem.
7. Solve: $\frac{dy}{dx} = \frac{y+2}{x-1}$.
8. Find the general solution of $y = (x - a)p - p^2$.
9. Solve: $(D^2 + 5D + 6)\,y = 0$.
10. Find the particular integral $(D^2 + 3D + 2)\,y = e^x$.

Part-B

Answer any FIVE questions: **(5 x 8 = 40)**

11. Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla\,.\,\vec{F}) - \nabla^2\vec{F}$.
12. Compute the divergence and curl of the vector $\vec{F} = xy^2\,\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$ at $(1, -1, 1)$.
13. Using Green's theorem, show that $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy = 20$, Where C is the boundary of the rectangular area enclosed by the lines $x = 0, x = 1, y = 0, y = 2$ in the xoy plane.
14. Evaluate $\iiint_V \nabla.\vec{F}dV$ where $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1$.
15. Solve: $p\,(1 + q^2) = q\,(z - 1)$.
16. Find the general solution of $(y + z)p + (z + x)q = x + y$.
17. Solve: $(D^2 + 5D + 6)y = e^x$.
18. Evaluate: $(D^2 + 16)\,y = cos4x$.

Part C

Answer any TWO questions: **(2 x 20 = 40)**

19. (a) Prove that $\vec{F} = (y^2\,cosx + z^3)\,\vec{i} + (2y\,sinx - 4)\,\vec{j} + (3x\,z^2)\vec{k}$ is irrotational and find it's scalar potential.

(b) Find the value of the integral $\int_C \overrightarrow{A} . d\vec{r}$ where $\overrightarrow{A} = yz\,\vec{\imath} + zx\,\vec{\jmath} - xy\,\vec{k}$ is the following cases (i) C is the curve whose parametric equation are $x = t,\ y = t^2,\ z = t^3$. Drawn from $(0, 0, 0)$ to $(2, 4, 8)$. (ii) C is the curve obtained joining $(0, 0, 0)$ to $(2, 0, 0)$ then $(2, 0, 0)$ to $(2, 4, 0)$ and then $(2, 4, 0)$ to $(2, 4, 8)$.

20. (a) Solve: $(x + 1)\ \frac{dy}{dx} + 1 = 2\, e^{-y}$.

(b) Solve: $\frac{dy}{dx} - y\, tanx = \frac{sinx\ cos^2 x}{y^2}$.

21. Verify Gauss divergence theorem for $\bar{F} = (2x - z)\,\bar{\imath} + x^2\, y\bar{\jmath} - xz^2\bar{k}$ over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

22. Solve $\frac{d^2y}{dx^2} + y = secx$, using variation of parameters.

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